Jet Noise at Low Reynolds Number

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There is considerable interest in the question of whether coherent structures that govern the mixing process in turbulent jets also affect the radiation of acoustic energy. This study indicates that below a Reynolds number of 10^5, where coherent structures are most easily observed, the spectral properties of jet noise change significantly. The relative acoustic power is lower in this Reynolds number range and the noise spectra scale with Helmholtz number rather than Strouhal number. There appear to be definite compressibility effects on the large-scale structure and flow development. Similar Mach number effects on the fine-scale turbulence are not noted.

Nomenclature

a = local speed of sound
a_0 = ambient speed of sound
df = bandwidth
D = jet diameter
f = frequency
He = Helmholtz number
k = wavenumber
M = Mach number
p = pressure
P_a = acoustic pressure
R = distance from jet to microphone
Re = Reynolds number
S = spectrum
St = Strouhal number
U = jet velocity
x = axial coordinate
y = radial coordinate
θ = angle between axis and microphone
ρ = density
ω = radian frequency

Introduction

OBSERVATIONS of large-scale wavelike structures in turbulent jets (e.g., Crow and Champagne[1]) lead to the question whether these coherent structures have an important impact on the sound radiation process. Flow visualization experiments readily identify the existence of coherent structures at Reynolds numbers below about 10^5. At higher Reynolds numbers an orderly motion is difficult to discern. This is due, in part, to the natural tendency for turbulent energy to be spread over a wider range of wavenumbers with increasing Reynolds number, in part, to the more rapid diffusion of tracers such as dye or smoke at higher velocities. The question remains whether the underlying coherent structure plays the same role in the acoustic radiation process as it becomes more deeply buried in background turbulence with increasing Reynolds number. This is an important point in view of the fact that most of the experimental evidence on coherent structures has been obtained at relatively low Reynolds number, whereas most acoustic data have been collected at relatively high Reynolds number.

Received May 20, 1982; revision received March 23, 1983.

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Normally, jet noise experimentation is conducted at Reynolds numbers well in excess of 10^5 (e.g., Lush[2] and Ahuja and Bushell[3]). Variations in Reynolds number are generally assumed to be inconsequential and Mach number is assumed to be the primary independent parameter. Hence, most of the available information on noise radiation vs Mach number have been obtained at variable Reynolds number by simply changing flow speed in a given size nozzle.

The forcing technique of Crow and Champagne[1] has been extended by Bechert and Pfizenmaier[4] and Moore[5] to include measurements in the acoustic far field. Here, forcing at frequencies in the range of the jet column instability ($0.2 < St_D < 0.8$) leads to a large increase in the level of broadband noise. On the other hand, excitation of low Reynolds number jets at much higher frequencies corresponding to the instability of the initial shear layer f_s produces noise amplification at f_s and its subharmonics (presumably through the mechanism of vortex pairing) and a reduction in the broadband level. Flow visualization indicates that forcing produces a high degree of coherence in the first few diameters of jet development, the motion being phase-locked to the excitation. The artificially created coherence is sensitive to both frequency and amplitude of the excitation. On the other hand, the naturally occurring coherent motions in low Reynolds number flow appear to be intermittent—jumping back and forth between an orderly structure and the chaotic motion with which turbulence is normally identified.[24]

This paper examines the noise radiation from low Reynolds number jets where coherent structure has been readily observed by other investigators. Because of the possibility that the Reynolds number may be an important independent variable, variations in Mach number are made at fixed Reynolds number and vice versa. Previously available information is scant. High subsonic Mach number at low Reynolds number requires the use of very small nozzles or a special low-density flow facility.[6] Typical exit diameters are about the size of the smallest conventional hot-wire probes and microphones, making a detailed survey of the turbulence next to impossible.

Experimental Methods and Data Reduction

Flow Conditions

The experimental program has been designed so that either the Reynolds number or the Mach number can be varied independently. Five different sized nozzles are used to obtain five different Reynolds numbers at a constant Mach number. Alternatively, Mach number can be varied at a constant Reynolds number. The flow conditions are shown on Fig. 1. The dots represent the flow conditions chosen. At any Mach number the Reynolds number can vary by almost 10. The heavy solid lines indicate the constraints imposed on the system. The horizontal line near the bottom was the original microphone sensitivity limit based on measured sound pressure levels (SPLs) obtained from the literature. It subsequently has been found that this estimate is too low. The spectra measured at $M=0.16$ showed that electronic and background noise dominated the signal at high frequencies, while excess noise from within the plenum chamber contaminated the mid and low frequencies. For this reason, measurements made at $M=0.16$ have not been used for further analysis. The system is limited to frequencies below 100 kHz due to microphone resonance.

Anechoic Chamber

The test facility is shown schematically in Fig. 2. The anechoic chamber at the St. Anthony Falls Hydraulic Laboratory (SAFHL) measures 2.2 m on a side and is lined with acoustic foam wedges 7-cm long, producing anechoic conditions above 1kHz. The chamber has a porous front wall to provide a uniform entrainment flow for the jet while shielding the chamber from spurious noises from the rest of the laboratory. The flow is drawn out of the room through a "jet catcher" at the rear wall and recirculated. The jet catcher is aligned on the same axis as the nozzles to further stabilize the flow.

The air for the jet is supplied from an air compressor. It passes through two filters, a pressure regulator, two mufflers, and into a 12.7-cm plenum chamber just upstream of the contraction. Manometers are used to monitor the total pressure within the plenum chamber. Exit Mach number is calculated from these measurements assuming isentropic flow.

Nozzles

All nozzles have an internal contour conforming to a fifth-order polynomial. This shape allows for the specification of slope and curvature to be zero at both the inlet and the outlet. Each of four small nozzles, 1.4, 2.4, 4.1, and 7.1 mm, attach onto a 25.4-mm preliminary contraction whereas a 12.2-mm nozzle is not preceded by a preliminary contraction. Justification for this design is based on experiments by Morel.[7] It was found that for contraction ratios greater than approximately 10 and length to diameter ratios of at least unity, the turbulence intensity at the exit is roughly independent of upstream conditions. All nozzles conform to these requirements: a length to diameter ratio of one and a contraction ratio greater than 16.

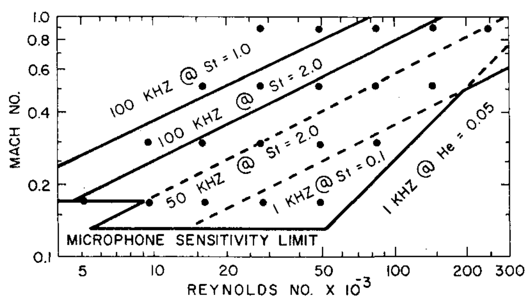

Fig. 1 Flow conditions.

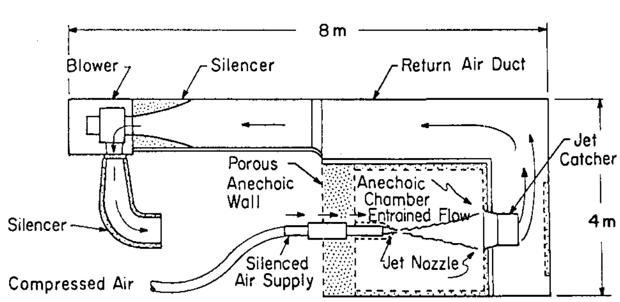

Fig. 2 Test facility.

Far Field Data Reduction

In order to correlate the measured spectra from different nozzles at different velocities and to compare these measurements with those from other investigations, it is necessary to render the data dimensionless. When expressing the true spectral density in terms of Strouhal number, the following definition is used.

$$S\left(\frac{fd}{U_j}\right) \equiv \frac{p_a^2}{(\rho U_j^2)^2}\left(\frac{R}{D}\right)^2\left(\frac{U_j}{\Delta f d}\right) \quad (1)$$

wherein ρ is the density of the jet flow at the nozzle exit plane, U_j the discharge velocity, R the distance to the observation point, D the jet diameter, and Δf the bandwidth. Similarly, the spectral density when correlated with Helmholtz number is expressed in the form

$$S\left(\frac{fd}{a_0}\right) \equiv \frac{p_a^2}{(\rho U_j^2)^2}\left(\frac{R}{D}\right)^2\left(\frac{a_0}{\Delta f D}\right) \quad (2)$$

wherein a_0 is the speed of sound in the *undisturbed* medium. In either case the amplitude of the normalized spectrum is proportional to acoustic efficiency. The variables ρ and U_j are determined from measured pressure and temperature in the plenum chamber assuming the flow to be isentropic and the pressure at the nozzle exit to be equal to the ambient pressure in the chamber. The experiments normally are performed at a stagnation temperature equal to room temperature which results in exit temperatures within the jet below ambient and densities within the jet that are higher than the surrounding undisturbed medium. Using ambient values for density and temperature would result in differences in the normalized spectrum by as much as 3 dB.

Because of the small jet diameters involved in this study, the frequency response of the measuring system must be very high. The 1/4-in. B&K microphones used have a resonant frequency of about 40 kHz. This necessitates certain corrections to the data for frequencies greater than about 10 kHz. One correction accounts for a resonant peak of about 12 dB. A second correction takes into account the size and geometry of the microphone relative to the wavelength and pattern of the sound field. Unfortunately, the second correction is a function of angle of incidence relative to the position of the noise source. The geometry of the microphone is designed such that at normal incidence (0 deg) the two corrections cancel, producing a relatively flat response curve to about 100 kHz.

Presentation of Results

Far Field Sound

Typical noise spectra obtained in the aft quadrant are shown in Fig. 3. These spectra appear to be similar to that observed at higher Reynolds number with the exception that the peak amplitude is centered at a lower Strouhal number, and the variation in peak frequency with angular position is not over a full octave as is generally accepted from the results of previous measurements. Most of the literature and textbooks (e.g., Goldstein[8]) present acoustic data in terms of ⅓-octave spectra. At 90 deg the generally accepted value of peak Strouhal number is 1.0, reducing to about 0.2-0.3 at 20 deg. This is not in agreement with the data shown in Fig. 3, and it was first assumed by the authors that this was a significant Reynolds number effect. However, when previously published ⅓-octave spectra are replotted as true power

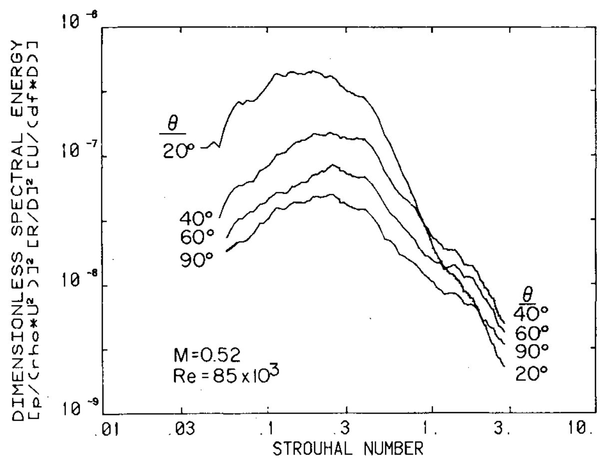

Fig. 3 Angular variation of narrowband spectral density.

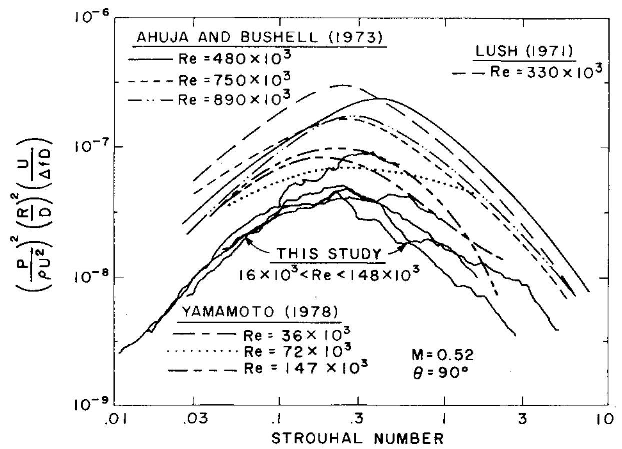

Fig. 4 Effect of Reynolds number on the spectral density at moderate Mach number, $M = 0.52$.

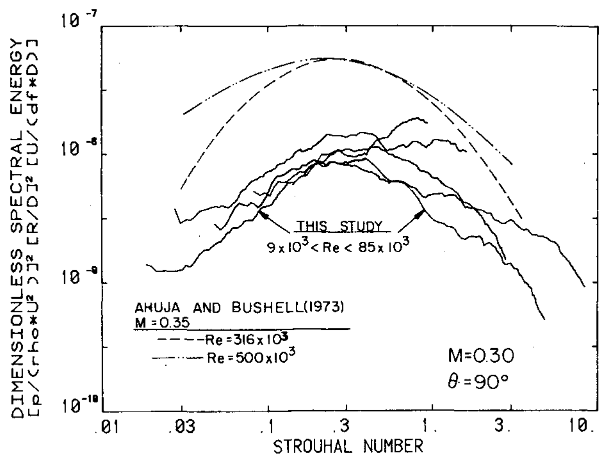

Fig. 5 Spectral density at low-Mach number, $M = 0.3$.

spectral density, the high Reynolds number data resemble that shown in Fig. 3, i.e., the well-known reverse Doppler shift practically disappears.

Figure 4 contains spectra at various Reynolds numbers and at a constant Mach number of 0.52. The data correspond to an angular position of 90 deg, and, thus, there is no uncertainty concerning possible Doppler corrections for convection. The most significant result is that there is no apparent variation with Reynolds number in the $16 \times 10^3 < Re < 148 \times 10^3$ range, but the noise level is significantly lower than that observed at higher Reynolds number by other investigators. Due to the surprising nature of this result the experiment was repeated, and the data double checked for accuracy. The results were the same; there is a difference of about 7 dB between low and high Reynolds number jet noise. This trend was also observed by Yamamoto[9] and was subsequently published by Yamamoto and Arndt[10] in a dimensionless form that showed a gradual shift between low and high Reynolds number experiments. However, the calculation was done incorrectly and when the original spectral curves were replotted, this gradual change was found not to exist. As shown on Fig. 4, the magnitude is about 2 dB above the result from the present experiment and 5 dB below that of high Reynolds number experiments. The nozzles used in Yamamoto's experiments had a simple circular arc contour. It is believed that the crude nature of the nozzle design produced a flowfield that is not as "clean" as the smoothly contoured fifth-order contraction used here.

The difference in relative spectral level of approximately 7 dB between the low Reynolds number experiments and those obtained at higher Reynolds number is significant and not related to the choice of the scaling parameters, density, and velocity. In making comparisons with other investigators, it was sometimes necessary to assume the value of the stagnation temperature and the ambient pressure. When this was necessary, the values corresponding to this study were assumed, i.e., 22°C and 740-mm Hg. A 5 deg change in stagnation temperature corresponds to a 2% change in density. However, as already mentioned, if the ambient density rather than the density of the jet flow was used, this could amount to as much as a 3 dB discrepancy. This is not a factor since the computed density at the given Mach number was always used in making comparisons.

The apparent lack of importance of Reynolds number in this range is unexpected. However, the remarks of Crighton[11] are noteworthy. He observes that all of the available acoustic data with forced jets fall into two categories according to whether the Reynolds number is less than or greater than 10^5. For $Re < 10^5$ excitation at or reasonably close to the spectral peak produces a response in the form of a broadband increase in noise level. For $Re < 10^5$ amplification is at the forcing frequency and its harmonics and subharmonics with a suppression of the broadband level. Crighton[11] points out that the differences in the two categories could be related to the initial conditions of flow development (laminar, transitional, or fully turbulent), the level of freestream turbulence, Mach number, and/or Reynolds number. In view of all the experiments on acoustic forcing of jets, it appears that the level of background noise surrounding the jet and from within the plenum may also affect the jet development significantly. Should this be true, the cutoff value of 10^5 for Reynolds number is only a nominal value and would vary with each test rig. This could explain the fact that our data display low Reynolds number effects to values as high as 2.5×10^5.

Attempts were made to determine why a 7-dB discrepancy exists between the amplitude of low Reynolds number jet noise spectra and the amplitude of high Reynolds number spectra. One possibility is that at low Reynolds number the boundary layer at the nozzle lip is laminar, and at high Reynolds number it is turbulent. Some initial experimentation was carried out to determine if the differences between laminar and turbulent boundary layers could result in a 7-dB difference in acoustic radiation. This was done by tripping the boundary layer with sand grain roughness. The results were inconclusive. Minor changes in the spectral characteristics were noted, but the overall level remained relatively unchanged.

Comparisons at lower Mach number are made in Fig. 5. Although it appears as if there is a significant difference in spectral level, it should be noted that the data of Ahuja and Bushell,[3] shown in the figure, are obtained at a Mach number that is 15% higher than the data obtained in this study. When

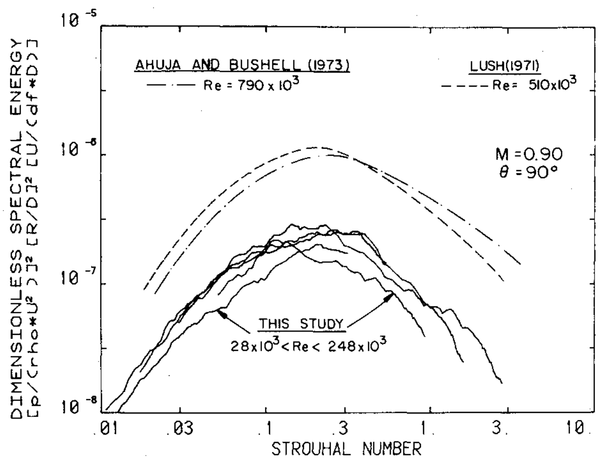

Fig. 6 Spectral density at high-Mach number, $M = 0.90$.

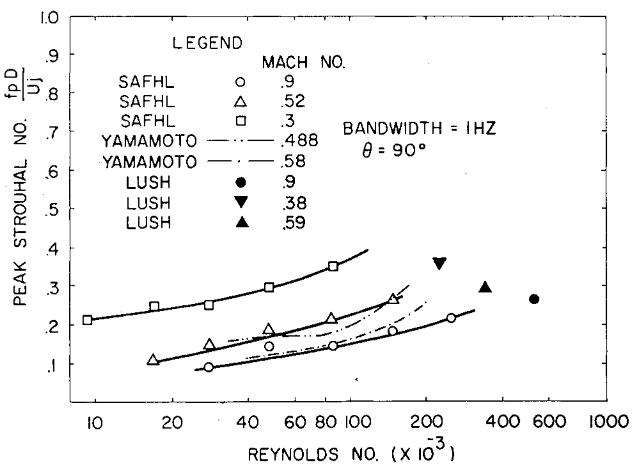

Fig. 8 Scaling of peak Strouhal number with Reynolds number at 90 deg.

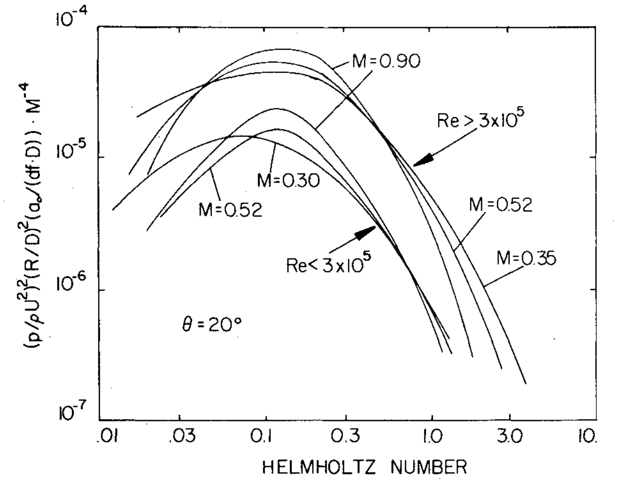

Fig. 7 Normalized spectral density eliminating Mach number variation by the U^8 law and using Helmholtz frequency scaling.

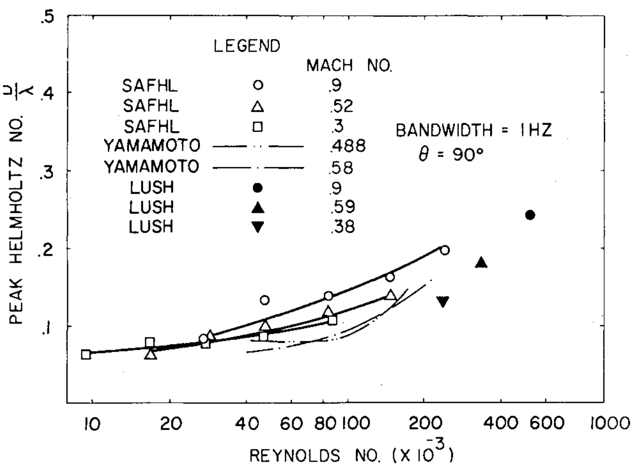

Fig. 9 Scaling of peak Helmholtz number with Reynolds number at 90 deg.

this is taken into account, there is no discernible difference in the spectral level. However, the discrepancy is still pronounced at higher Mach number as shown in Fig. 6. This lends further credence to the idea that the jet development is a complex function of Reynolds number, initial shear layer characteristics, and freestream turbulence level as well as Mach number. Similar trends are observed at other positions relative to the jet axis.

An important point is that the data are presented with no corrections for Doppler shift. In fact, if the data were so corrected, even greater differences in the peak Strouhal number would be observed. Helmholtz number appears to be a better frequency scaling parameter. This is shown in Fig. 7 where the various Mach number data collected at $\theta = 20$ deg are collapsed according to a M^5 scaling law. (The M^5 dependence on acoustic efficiency is accounted for by dividing by M^4 since the characteristic frequency for Helmholtz scaling is a_0/D rather than U/D.) Reasonable collapse of the data is experienced, but there are still variations in spectral shape with Mach number. The variation with Reynolds number is evident. Keep in mind that each curve in Fig. 7 is the average of five or six spectra. Further evidence supporting the use of Helmholtz number as a scaling parameter is shown in Figs. 8 and 9. Comparisons are made of the peak frequency vs Reynolds number at constant Mach number using Strouhal scaling and Helmholtz scaling. Comparisons are made at 90 deg to eliminate any ambiguity of Doppler shift. It is evident that better collapse of these data is accomplished using Helmholtz scaling. It is generally accepted that Helmholtz scaling may be more appropriate at radiation angles close to the jet axis. However, the fit to the sideline data is unexpected and could be a significant result. The significance of Helmholtz scaling implies that the sound sources within the jet acts like a distribution of stationary sources. This point is underscored by the results of Kibens,[12] who found that excitation of a jet at the shear layer instability frequency f_s produced amplification in the acoustic field at f_s and its subharmonics *without any Doppler shift* implying that the sound sources were stationary.

Near Field Pressure

The sizes of the jets used in this investigation are prohibitively small for any detailed examination of the turbulence with hot wire anemometry. However, further information about the nature of the sound sources can be gleaned from measurements in the acoustic near field. Comparisons of near field spectra at different Reynolds number are shown in Fig. 10. No discernible variation with Reynolds number is noted except at high frequencies. At high frequencies ($St \geq 1.0$) the microphone is actually in the far field. On a wavelength basis this corresponds to being more than two wavelengths from the center of the mixing layer. At this distance it can be shown that the propagating part of the disturbance caused by an isolated acoustic quadrupole outweighs the near field nonpropagating part. It is important to note that the near field pressure spectrum in the inertial subrange decays more rapidly with an increase in frequency than is observed for pressure fluctuations measured within the

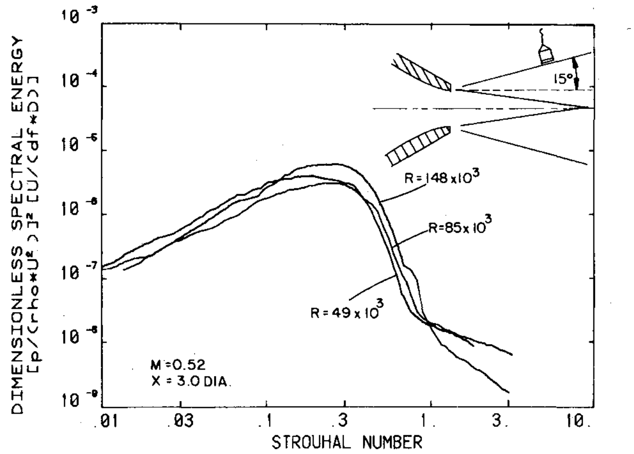

Fig. 10 Near field pressure spectra at low-Reynolds number and moderate Mach number, $M = 0.52$.

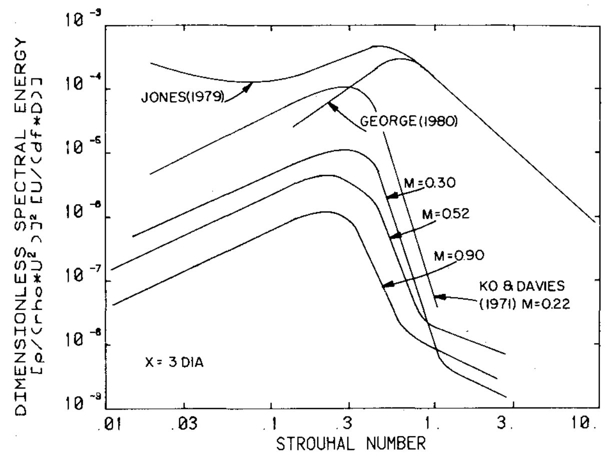

Fig. 12 Variation of near field pressure spectrum with Mach number. Each curve represents an average over all Reynolds numbers tested.

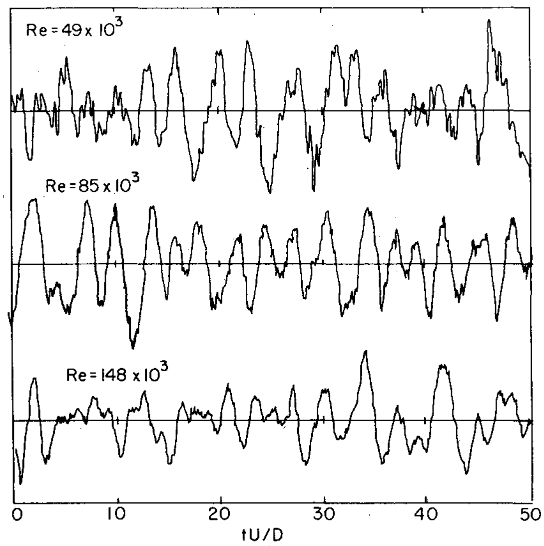

Fig. 11 Near field pressure signal at various Reynolds numbers and constant Mach number, $M = 0.52$.

jet mixing region. The decay is much steeper than the $k^{-7/3}$ variation first suggested by Batchelor.[13] George et al.[14] and Jones et al.[15] both show that the resulting pressure spectra are due to two basic terms first proposed by Kraichnan[16]: a turbulence-turbulence term which rolls off as $K^{-7/3}$ and a turbulence-mean shear effect which has a $k^{-11/3}$ decay. However, the observed decay in the inertial subrange on the order of k^{-8} is much steeper than that predicted by turbulence theory alone. The large discrepancy can be qualitatively accounted for by considering an isolated acoustic quadrupole, whose near field strength is proportional to $\rho_0 a_0 u(kR_0)$ where the velocity u is assumed proportional to the turbulence intensity. Further, the product of wavenumber and source size, kR_0, is assumed constant, i.e., the frequence of sound generated is inversely proportional to source size. The fluctuations in mean square pressure in the near field die off as $(kr)^{-6}$. If r is held constant, as it is for the spectra shown in Figs. 10 and 12, the spectral decay is k^{-6}. This k^{-6} weighting coupled with the decay due to the turbulence of $k^{-5/3}$ yields a value similar to that observed.

Although the near field pressure spectra are relatively insensitive to Reynolds number, some differences can be discerned in the raw pressure signal. This is illustrated in Fig. 11. Pressure as a function of normalized time Ut/D is plotted for three different Reynolds numbers at a fixed Mach number of 0.52. The period of the primary disturbance remains fixed, but at higher Reynolds number the primary periodicity is interrupted relatively more frequently. This is readily evident when comparing the highest Reynolds number signal with the lowest Reynolds number signal. At high Reynolds number, scarcely two periods can elapse without interruption. At low Reynolds number as many as five periods can elapse in a relatively coherent manner. If each primary period is assumed correlated with the passage of a coherent structure, one gets a picture in qualitative agreement with Lau and Fisher.[17] However, simultaneous measurement of near field pressure and flow visualization by Sarohia and Massier[18] do not confirm this concept. They could find no correlation between a peak in the near field pressure signal and the passage of a large structure. The pressure signal is the result of eddies being convected past a fixed microphone. At high Reynolds number the range of eddy sizes is wide and the large structure is more deeply buried in the fine-grained structure making the detection of the passage of a large structure less likely. At low Reynolds number the opposite is true and the passage of a large structure should be more readily observed. This provided a crude explanation of the differences in the signals plotted in Fig. 11.

The Mach number variation of the near field pressure is shown in Fig. 12. The curves shown represent average values since variation with Reynolds number is minimal. Comparison is made with pressure measurements within the jet mixing layer by George et al.[14] and Jones et al.[15] Also shown are near field pressure measurements by Ko and Davies[19] at the same relative location but at a lower Mach number. There is a significant reduction in the relative pressure amplitude with increasing Mach number.

The pressure sensed by a microphone outside of the jet reflects the global average of all the disturbances within the jet, including spatial decay. If consideration is given to the fact that the spreading rate of the jet decreases with increasing Mach number and that hydrodynamic pressure dies off like R^{-3}, the decrease in pressure level can be accounted for (at least qualitatively) by arguing that the relative distance from the microphone to the origin of the pressure disturbances increases with increasing Mach number.

Discussion

Mollo-Christensen and Narasimha[20] appear to be the first to present spectra in the dimensionless form given in Figs. 3-6. The Reynolds number range ($Re < 2 \times 20^5$) in their work is also comparable to that in this study. Unfortunately, attempts to make quantitative comparisons of the spectra in the two studies were without success. However, their conclusions are in agreement with the results of this study. Specifically, they concluded that Reynolds number is an important parameter, that Helmholtz scaling was somewhat better than Strouhal scaling, and that the acoustic efficiency scaled with the fourth to sixth power of Mach number depending on the radiation angle. The results of this investigation indicate an effect of Reynolds number, but not the gradual effect previously proposed. A sharp break at approximately $Re = 10^5$ is observed. As noted by Crighton,[11] factors other than the initial thickness of the shear layer (which was assumed to be the primary Reynolds number by Mollo-Christensen and Narasimha[20]) have an impact on jet flow development and subsequent sound radiation. Crighton's survey of the published experiments with forced jets indicates that forcing enhances the coherency of the motion in all cases. At low Reynolds number the motion is quite regular and is locked in phase with the excitation signal. This enhanced coherency is apparently at the expense of the fine-grained turbulence which becomes less energetic. This implies that the coherent structure now contains energy which in the unexcited case would be found in the smaller eddies. Kibens'[12] results clearly demonstrate this concept. Upon excitation, the broadband pressure fluctuations in both the far field and near field of a low-Reynolds number jet decrease whilst the pressure amplitude at the excitation frequency and its subharmonics increase dramatically. A net increase in OASPL is experienced since the increase in energy at discrete frequencies outweighs the decrease in the broadband level. A different process is observed when a high-Reynolds number jet is excited. The energy level in the coherent structure does not appear to be at a much higher level than would exist naturally. The structure, however, is more organized. In this case the broadband sound level increases. The only discrete tone that can be observed in the spectrum is at the excitation frequency. Apparently, the observation of a sharp difference in the broadband sound level of unforced high and low Reynolds number jets is related to the observed differences in response to excitation at high and low Reynolds numbers. Why the response is different is yet to be explained. A qualitative argument for differences in the sound radiation after forcing can be made on the basis of vortex interaction. In a low Reynolds number forced jet, interaction occurs mainly with eddies of like size. Without excitation the interaction tends to occur over a broader range of eddy scales. Intuitively one would expect that interaction of eddies of similar size would be more violent than interactions between different sized eddies. Thus, it can be argued that, at low Reynolds number, vortex pairing of the large-scale structure is enhanced at the expense of interaction between the large scales and fine-grained turbulence. Since the Lighthill source term is quadrupole in nature, it is necessary to examine the pairing process to determine if it contains the necessary ingredients for noise radiation. Ffowcs Williams and Kempton[21] argue that the pairing process is rapid enough for the sound radiation to be significant. This leads to the argument that, at low Reynolds number, the pairing process in a forced jet can have a dominant effect on the noise radiation. This is consistent with the observation of Browand and Weidman[22] that a significant Reynolds stress is produced by each pairing event. Without pairing, a large eddy suffers little distortion over its lifetime.[23]

At high Reynolds number, the coherent structure can be made more evident by excitation, but it can be conjectured that the dominant noise source is due to interaction between the large-scale structure and the fine-grained turbulence. In this case external excitation apparently enhances this interaction and increases the broadband level without any apparent enhancement of noise due to pairing.

Conclusions

An extensive survey of the spectral characteristics of the far field noise has revealed no effect of Reynolds number in the range $16 \times 10^3 < Re > 250 \times 10^3$. However, the noise level is found to be about 7 dB lower than high Reynolds number experiments ($Re > 2 \times 10^5$) carried out at other facilities. Consideration of this apparent discrepancy leads to the conclusion that although Reynolds number appears to be the decisive parameter, other factors must be considered, particularly the initial conditions at the jet exit.

Correlation of frequency dependent data is made without the Doppler correction, $(1 - U_c \cos\theta)$. Preliminary analysis of the data appears to indicate that this correction would be inappropriate, implying that the sound sources act as though they were stationary. Furthermore, the data appear to correlate better with Helmholtz number ($fd/a_0 = d/\lambda$) than with Strouhal number (fd/U_j).

Normalized near field pressure spectra *decrease* in amplitude with *increasing* Mach number as opposed to far field noise which increases with Mach number. This may indicate that there is not a significant effect of Mach number on the noise producing fine-grained turbulence, but that there is a compressibility effect on the nonpropagating, irrotational pressure field surrounding the jet.

It is conjectured that vortex pairing (or large-scale interaction) can only dominate the noise process at low-Reynolds number because at high Reynolds number it is buried beneath the noise produced by large-scale/small-scale interactions. This is concluded from the observation that large-scale coherent structures are easily identifiable at low Reynolds number and that excitation in this range produces significant tones at the excitation frequency and its subharmonics. Similar excitation of high Reynolds number jets only increases the broadband level.[11]

Acknowledgments

The authors are grateful for the assistance of the Air Force Office of Scientific Research under Contract F49620-80-C-0053.

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